Bayes to the Rescue

Markov Chain Monte Carlo in the pMSSM

Mike Saelim

Laboratory for Elementary Particle Physics
Cornell University

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Outline

1. Motivation: Scanning the pMSSM
2. Bayesian Statistics
3. Markov Chain Monte Carlo
4. Pretty Pictures

Motivation: Scanning the pMSSM

- pMSSM = phenomenological Minimal Supersymmetric Standard Model
  - EWSB-scale model with 22 soft parameters
  - First and second generations degenerate

3 gaugino masses:	M_1, M_2, M_3
4 slepton masses:	$m_{e_L}, m_{\tau_L}, m_{e_R}, m_{\tau_R}$
6 squark masses:	$m_{qu_L}, m_{Q_L}, m_{u_R}, m_{d_R}, m_{t_R}, m_{b_R}$
6 trilinear couplings:	$A_e, A_\tau, A_u, A_d, A_t, A_b$
3 Higgs sector parameters:	M_A (pole), $\tan\beta\,(m_Z)$, μ

- Future scenario: measurements have picked out a benchmark point (BP) in the pMSSM.
- Given the values and uncertainties of these measurements,
  - What is our uncertainty in the benchmark point?
  - What are the predicted values and theoretical uncertainties for not-yet-measured quantities?

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Motivation: Scanning the pMSSM

Okay... how?

- Scan around the BP to figure out how likely each point is, given the measurements
- Traditional method: grid scan! But...
  - ...in order to get a fine enough resolution, you'll be scanning for years.
- Solution: Markov Chain Monte Carlo
  - E.A. Baltz, M. Battaglia, M. Peskin, T. Wizansky (hep-ph/0602187)
  - S.S. AbdusSalam, B.C. Allanach, F. Quevedo, F. Feroz, M. Hobson (0904.2548)
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Bayesian Statistics

- I want the Bayesian posterior probability density function (PDF)
  - The probability of each point in the pMSSM being true after being “confronted” with “evidence”

Bayes’ Theorem

For parameters X and data D,

$$\text{Posterior PDF}(X|D) \sim \mathcal{L}(X|D) \times \text{Prior PDF}(X)$$

- In a way, all of science works this way
- This has also led to Bayesian Search Theory, which has been used to recover
  - lost submarines (*USS Scorpion*)
  - lost oil tankers (*MV Derbyshire*)
  - lost historical ships (*SS Central America*)
  - lost undetonated fusion warheads (Palomares incident)

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Markov Chain Monte Carlo

Adaptive Metropolis-Hastings Algorithm (Baltz et al.)

- Method for scanning a parameter space by confronting a prior PDF with new evidence, producing a posterior PDF

- Markov Chain $\longrightarrow$ the next point is chosen only based on the current point
- Monte Carlo $\longrightarrow$ words you can add to anything involving randomness

- The chains spread out and randomly explore the parameter space around the BP
- Eventually, the distribution of points converges to the posterior PDF, independent of the choice of prior PDF ‡
- We can also calculate the distribution of experimental observables over the posterior PDF

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∗ Likelihood function

For experimental measurements $\{M_i\}$ and measurement data $\{(m_i, \sigma_i)\}$,

$$\mathcal{L}(X) = \prod_i \exp\left[-\frac{(M_i(X) - m_i)^2}{2\sigma_i^2}\right]$$

† Choosing a trial point

The trial point is chosen from a Gaussian distribution with an adaptive covariance matrix:

$$P(\vec{y}) \sim \exp\left[-\frac{1}{2}\vec{y}^T C^{-1} \vec{y}\right]$$

‡ Convergence

Convergence algorithm given in Dunkley et al. (astro-ph/0405462)

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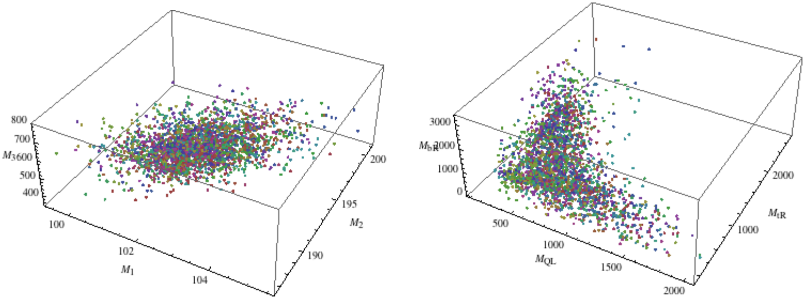
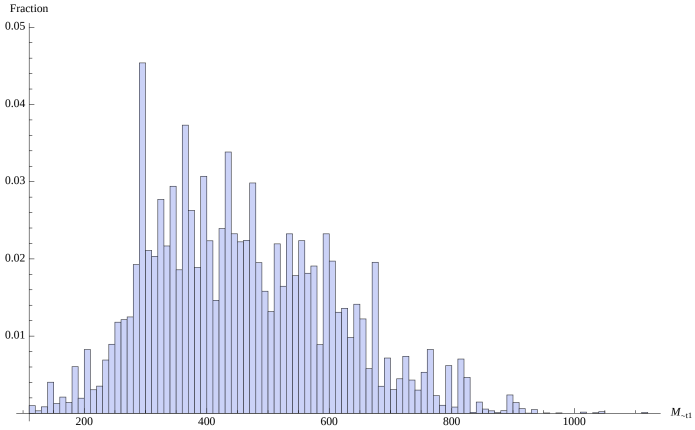
Pretty Pictures

- 11-dimensional subspace, 50 chains, 1M points $\longrightarrow$ 2 days on the Macbook
- Area surrounding benchmark point LCC1 with predicted uncertainties from ILC @1TeV (Baltz et al.)
- Particle spectra calculated with SuSpect (hep-ph/0211331)



Pretty Pictures



I have run out of things to say that can hold your attention

- Markov Chain Monte Carlo is a simple algorithm for generating posterior PDFs
- Posterior PDFs can then be used to generate distributions for not-yet-measured quantities
- MCMCs are especially useful on large-dimensional parameter spaces such as the pMSSM